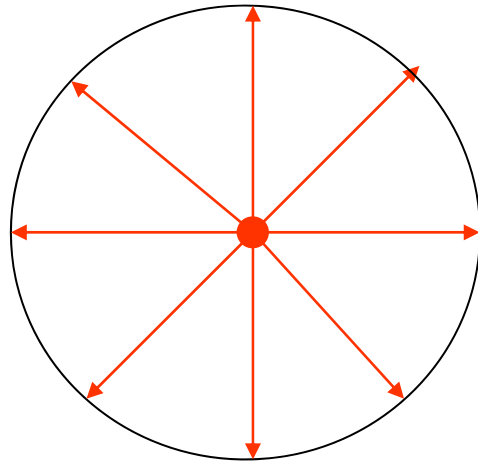


Lect. 6: Interference

Consider isotropic EM wave radiation by a point source.

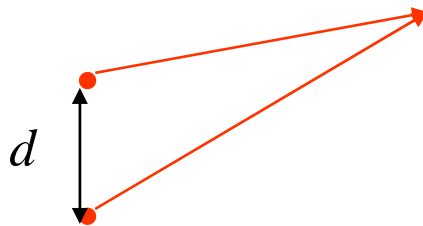


$$E \sim \frac{1}{R} e^{-jkR} \text{ (Spherical wave)}$$

Why $\frac{1}{R}$ dependence?

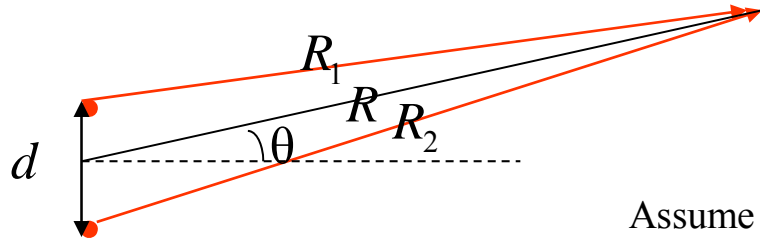
because $\int |E|^2 R^2 \sin \theta d\theta d\phi$ should be constant.

Two point sources separated by d



$$E_{\text{total}} = ?$$

Lect. 6: Interference



$$E(R, \theta, \phi) = E_1 + E_2$$

$$= \frac{A}{R_1} e^{-jkR_1} + \frac{A}{R_2} e^{-jkR_2}$$

Assume $R \gg d$ $R_1 \approx R - \frac{d}{2} \sin \theta$ and $R_2 \approx R + \frac{d}{2} \sin \theta$

$$\therefore E(R, \theta) \approx \frac{A}{R} e^{-jk(R - \frac{d}{2} \sin \theta)} + \frac{A}{R} e^{-jk(R + \frac{d}{2} \sin \theta)}$$

$$= \frac{A}{R} e^{-jkR} (e^{jk\frac{d}{2} \sin \theta} + e^{-jk\frac{d}{2} \sin \theta}) = \frac{2A}{R} e^{-jkR} \cos(k\frac{d}{2} \sin \theta)$$

$I(\text{Intensity}) \sim |E|^2 = 4(\frac{A}{R})^2 \cos^2(k\frac{d}{2} \sin \theta) \Rightarrow$ There exist max. and min. intensity conditions:

For max., $k\frac{d}{2} \sin \theta = m\pi \Rightarrow E = \frac{2A}{R}$;

Phase difference = $2m\pi$; In Phase

\rightarrow Constructive interference

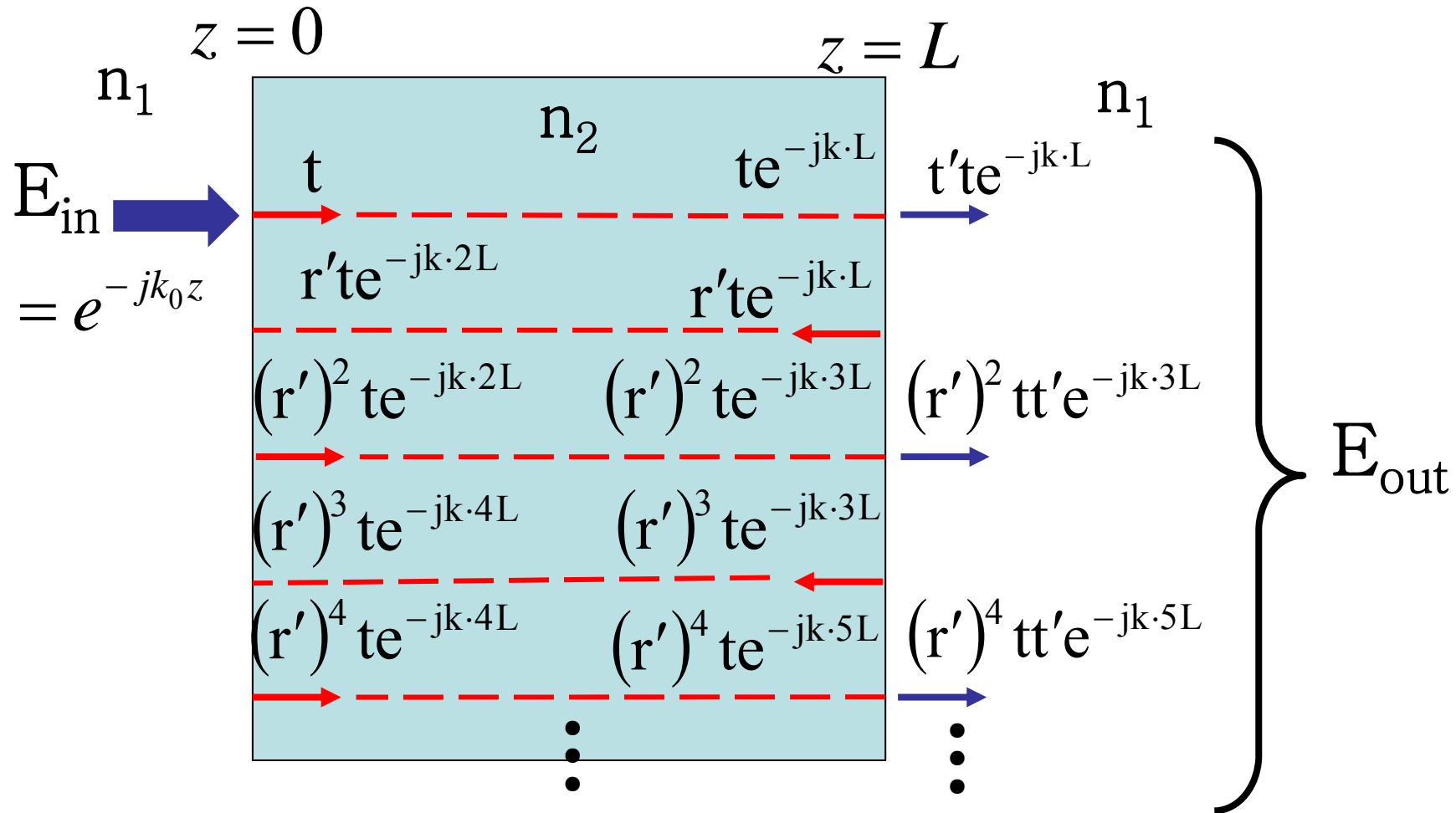
For min., $k\frac{d}{2} \sin \theta = (m + \frac{1}{2})\pi \Rightarrow E = \frac{A}{R} - \frac{A}{R} = 0$;

Phase difference = $(2m + 1)\pi$; Out of Phase

\rightarrow Destructive interference

Lect. 6: Interference

Interference in a dielectric slab (Fabry-Perot Interferometer)

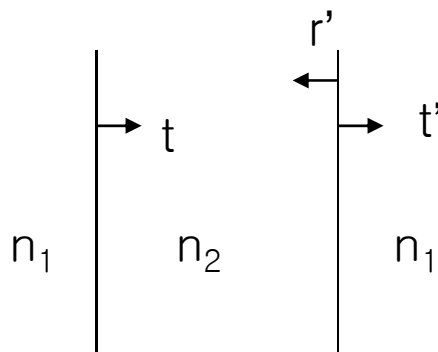


Lect. 6: Interference

Interference in a dielectric slab (Fabry-Perot Interferometer)

$$E_{out} = E_{t,total} = tt'e^{-jk \cdot L} + (r')^2 tt'e^{-jk \cdot 3L} + (r')^4 tt'e^{-jk \cdot 5L} + \dots = \frac{tt'e^{-jk \cdot L}}{1 - (r')^2 e^{-j2kL}}$$

$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1 - (r')^2 e^{-j2kL}][1 - (r')^2 e^{j2kL}]} = \frac{(tt')^2}{[1 - (r')^2]^2 + 4r'^2 \sin^2(kL)}$$



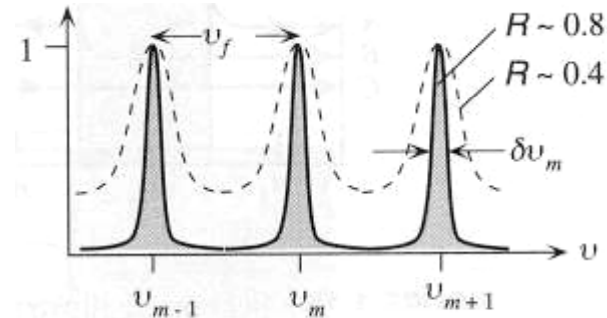
$$r' = \frac{n_2 - n_1}{n_2 + n_1}, \quad t = \frac{2n_1}{n_1 + n_2}, \quad t' = \frac{2n_2}{n_1 + n_2}$$

$$\therefore tt' = \frac{4n_1 n_2}{(n_1 + n_2)^2}, \quad 1 - r'^2 = \frac{4n_1 n_2}{(n_2 + n_1)^2}$$

$$\text{Let } R = r'^2, \quad T = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(kL)}$$

Lect. 6: Interference

$$T = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(kL)}$$



Max. Transmission: $\sin(kL) = 0 \Rightarrow T = 1$

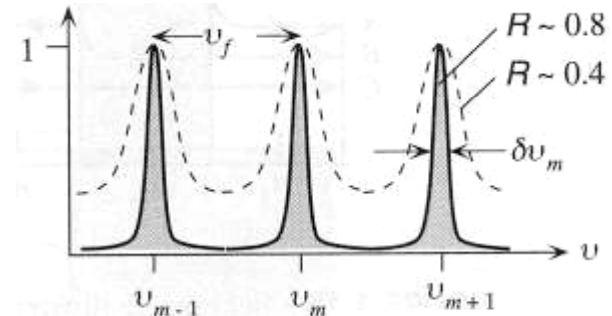
$$kL = m\pi; n_2 \frac{2\pi}{\lambda} L = m\pi \Rightarrow L = m \frac{\lambda}{2n_2} \text{ (Half-wave)}$$

Min. Transmission: $\sin(kL) = 1$

$$kL = (m + \frac{1}{2})\pi; n_2 \frac{2\pi}{\lambda} L = (m + \frac{1}{2})\pi \Rightarrow L = \frac{\lambda}{2n_2} (m + \frac{1}{2}) \text{ (Quarter-wave)}$$

Lect. 6: Interference

$$T = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(kL)}$$



Period? → Free Spectral Range

$$\Delta kL = \pi \Rightarrow \Delta k = \frac{\pi}{L}$$

$$\Delta \omega = ? \quad \text{Since } k = n_2 \frac{\omega}{c}, \quad \Delta \omega = \frac{c}{n_2} \Delta k = \frac{c}{n_2} \frac{\pi}{L} \quad \Delta f = \frac{c}{2n_2 L} = \frac{1}{T};$$

$$T = \frac{2L}{c/n_2}; \text{ round-trip time}$$

$$\Delta \lambda = ? \quad \lambda = n_2 \frac{2\pi}{k} \quad \Delta \lambda = \frac{d\lambda}{dk} \Delta k = -n_2 \frac{2\pi}{k^2} \Delta k = -\frac{\lambda^2}{2n_2 L}$$

Lect. 6: Interference

$$T = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(kL)}$$

Sharpness?

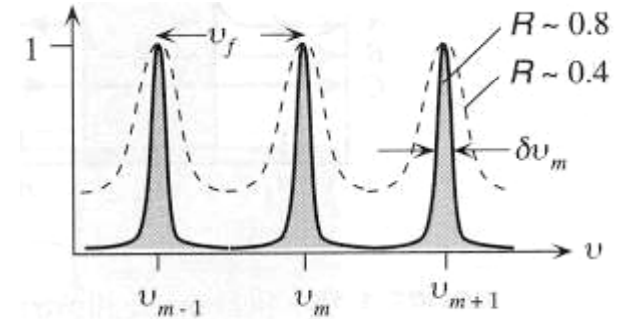
Determine k where $T = 0.5$

$$\frac{1}{2} = \frac{1}{1 + \frac{4R}{(1 - R)^2} \sin^2(kL)} \quad \text{or} \quad \frac{4R}{(1 - R)^2} \sin^2(kL) = 1$$

$$kL = \sin^{-1} \sqrt{\frac{(1 - R)^2}{4R}} = \sin^{-1} \frac{(1 - R)}{2\sqrt{R}}$$

$$\text{FWHM (Full Width at Half Maximum)} = 2 \sin^{-1} \frac{(1 - R)}{2\sqrt{R}} \sim \frac{(1 - R)}{\sqrt{R}} \quad (\text{If FWHM} \ll 1)$$

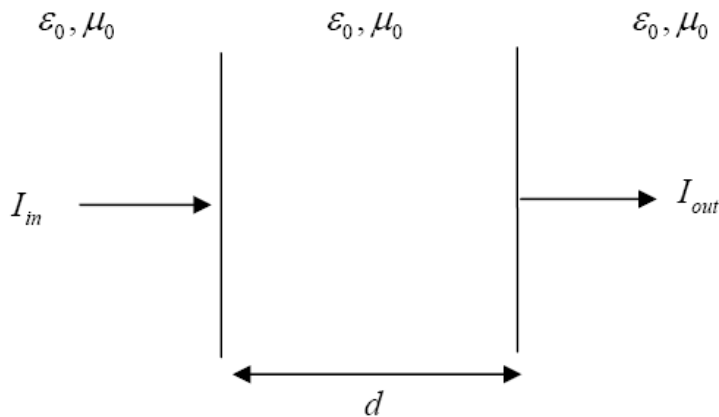
R increases, FWHM decreases => sharper response



Lect. 6: Interference

Homework:

Consider a Fabry–Perot interferometer made of two identical partially reflecting/transmitting mirrors as shown below. The mirrors transmit and reflect half of the incident power. (Or it has $r = 1/\sqrt{2}$ and $t = j/\sqrt{2}$).



- What is I_{out}/I_{in} ? Give your answer as a function of $\sin(kd)$, where k is the wavenumber in the vacuum and d is the distance between two mirrors.
- If the output power is plotted as a function of the frequency of the input light, what is the frequency separation between two adjacent peaks? Express your answer in terms of c , speed of light, d , and other fundamental parameters if required.
- What is the finesse of this interferometer? Give a numerical answer.